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DOUBLE EDGE-VERTEX DOMINATION UNDER SOME GRAPH OPERATIONS

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Abstract. An edge $e = uv$ of graph $G = (V, E)$ is said to ev-dominates vertices u and v as well as all vertices adjacent to u and v. A set $S \subseteq E$ is double edge-vertex dominating set, if every vertex of V is ev-dominated by at least two edges of S. The minimum cardinality of a double edge-vertex dominating set of G is the double edge-vertex domination number and is denoted $\gamma_{dev}(G)$ which was defined in Kılıç & Aylı (2020). In this paper, we give formulas for some corona products and some cartesian products of graphs on double edge-vertex domination number.

Keywords: Edge-vertex dominating set, vertex-edge dominating set, double edge-vertex dominating set, double vertex-edge dominating set.

AMS Subject Classification: 68R10, 68M10, 05C69.

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1 Introduction

Let $G = (V, E)$ be a simple graph. The set $N(v) = \{v \in V | uv \in E\}$ is the open neighborhood and $N[v] = N(v) \cup N(u)$ is the closed neigboorhood (Kulli, 2016) of $v \in V$. For any edge $e \in E$, the open edge neighborhood (Kulli, 2015) $N(e)$ of e is the set of edges adjacent to e. For $S \subseteq V$ of vertices in a graph $G = (V, E)$ is called a dominating set if every vertex $v \in V$ is either an element of S or adjacent to an element of S. The minimum cardinality of a dominating set of G is called the domination number (Haynes et al., 1998) and is denoted by $\gamma(G)$. A subset X of E is called an edge dominating set of G if every edge not in X is adjacent to some edge in X. The edge domination number $\gamma^{'}(G)$ of G is the minimum cardinality taken over all edge dominating sets of G. The concept of edge domination established by Mitchell & Hedetniemi (1977).

A vertex v of $G = (V, E)$ is said to ve-dominate every edge incident to v, as well as every edge adjacent to these incident edges. A set $D \subseteq V$is vertex-edge dominating set, if every edge of E is ve-dominated by at least one vertex of D. The minimum cardinality of a vertex-edge dominating set of G is the vertex-edge domination number and is denoted by $\gamma_{ve}(G)$. The concept of vertex-edge domination was first defined by Peters (1986). A set $D \subseteq V$ is a double vertex-edge dominating set of G if every edge of E is vertex-edge dominated by at least two vertices of D. The double vertex-edge domination number of G, denoted by $\gamma_{dve}(G)$, is the minimum cardinality of a double vertex-edge dominating set of G. A double vertex-edge dominating set of G of minimum cardinality is called a $\gamma_{dve}(g)$-set. Double vertex edge-domination concept was introduced by Krishnakumari et al. (2017).

An edge $e = uv$ of graph $G = (V, E)$ is said to edge-vertex dominate vertices u and v as well as all vertices adjacent to u and v. A set $S \subseteq E$ is an edge-vertex dominating set, if every vertex of V is ev-dominated by at least one edge of S. The minimum cardinality of an edge-vertex dominating set of G is the edge-vertex domination number and is denoted by $\gamma_{ev}(G)$. Edge-vertex dominating set and edge-vertex domination number were defined by Lewis (2007).

A subset $S \subseteq E$ is a double edge-vertex dominating set, if every vertex of V is ev-dominated by at least two edges of S. The minimum cardinality of a double edge-vertex dominating set of G is the double edge-vertex domination number and is denoted by $\gamma_{dev}(G)$ in Kılıç & Aylı (2019) and Kılıç & Aylı (2020).

Graphs are powerfull structures in representing problems involving objects and their relationships. Graph operations have a wide range of usage in real life problems. These are used to obtain complex structures in modeling network systems. Two of important graph operations are corona and cartesian processes. In this paper, we find double edge vertex domination number of some graphs under corona and cartesian products operation.

2 Double Edge Vertex Domination of Corona Products of Some Graphs

In this section, we find results on double edge-vertex domination number of corona products of some graphs and we prove the results.

Definition 1. *Let G_1 and G_2 be two graphs of sizes p_1 and p_2 respectively. The corona $G_1 \odot G_2$ is defined as G which is obtained by taking one copy of G_1 and p_1 copies of G_2, and then joining the i'th node of G_1 to every node in the i'th copy of G_2 (Buckley & Harary, 1990).*

Theorem 1. *For a path with order $n \geq 2$ and any graph G with order m, double edge-vertex domination number of cartesian products of these are as follows,*

$$\gamma_{dev}(P_n \bigodot G_m) = n + 1$$

Proof. Let P_n be a path and G_m be any graph, $V = \{v_1, v_2, ..., v_n\}$ be set of vertices of P_n and $E = \{e_1, e_2, .., e_{n-1}\}$ be set of edges of P_n respectively. The edges with the largest neighborhood are the edges of P_n, since we join the i.th vertex of P_n to each vertex in the i.th copy of G_m. So each inner edge of P_n has $2m+2$ edges in neighboorhood and pendant edges has $2m+1$ edges in neighboorhood. Therefore, we start to select elements of double edge-vertex dominating set from the edges of P_n. The edge e_1 edge-vertex dominates all vertices once which is adjacent to the vertex v_1 and which is adjacent to the vertex v_2. If we continue with e_2, also e_2 edge-vertex dominates all vertices which is adjacent to the v_2 vertex and which is adjacent to the vertex v_3. Thus, the vertices adjacent to the vertex v_2 are dominated twice. If we continue to select sequentially in this way until the last edge e_{n-1}, then the vertices of G_m joined to v_1 and the vertices of the P_n joined to v_n are both dominated once, while all other vertices are dominated twice. So, we selected $n-1$ edges upto now. If we select one edge from each G_m joined to the vertices v_1 and v_2, all vertices will be dominated twice. So, we select $n-1+2 = n+1$ edges to double edge-vertex dominate all vertices in resulting graph. Then we have

$$\gamma_{dev}(P_n \bigodot G_m) = n + 1$$

. □

Theorem 2. *For a cycle with order $n \geq 3$ and any graph G with order m, double edge-vertex domination number of cartesian products of these are as follows,*

$$\gamma_{dev}(C_n \bigodot G_m) = n$$

Proof. Let C_n be a cycle and G_m be any graph of order m, $V = \{v_1, v_2, ..., v_n\}$ be the vertex set of C_n and $E = \{e_1, e_2, .., e_n\}$ be the edge set of C_n. The edges with the largest neighborhood are the edges of C_n, since we join the i.th vertex of C_n to each vertex in the i.th copy of G_m. So each edge of C_n has $2m+2$ edges in neighboorhood. Therefore, we start to select elements of double edge-vertex dominating set from the edges of C_n. The edge e_1 edge-vertex dominates all vertices once which is adjacent to the vertex v_1 and which is adjacent to the vertex v_2. If we continue with the edge e_2, also e_2 edge-vertex dominates all vertices which is adjacent to the vertex v_2 and which is adjacent to the vertex v_3. Thus, the vertices adjacent to v_2 are dominated twice. v_i and v_{i+1} are endpoints of e_i, v_{i+1} and v_{i+2} are endpoints e_{i+1}. With each selection of e_i and e_{i+1}, each of these two edges dominates vertex v_{i+1} once which is one of the common endpoints of e_i and e_{i+1}. So, vertex v_{i+1} is dominated twice. If we continue in this way until the last edge e_n, then all vertices will be edge-vertex dominated twice. So, we selected n edges of C_n to double edge-vertex dominate all vertices. Then we have

$$\gamma_{dev}(C_n \bigodot G_m) = n.$$

□

3 Double Edge-Vertex Domination of Cartesian Products of C_n with P_2 and P_3

In this section, we find results for double edge-vertex domination number of cartesian products of C_n with P_2 and P_3 and we prove that these results.

Definition 2. *The cartesian product of graphs G and H denoted as $G \bigotimes H$ with vertices $V(G \bigotimes H) = V(G) \bigotimes V(H)$ and $(a,x)(b,y)$ is an edge of $G \bigotimes H$ if $a = b$ and $xy \in (H)$, or $ab \in (G)$ and $x = y$ (Imrich et al., 2008).*

Theorem 3. *For a cyle with order $n \geq 3$ and path with order 2, double edge-vertex domination number of corona products of these are as follows,*

$$\gamma_{dev}(C_n \bigotimes P_2) = 2\lfloor\frac{n}{3}\rfloor + n \ mod\ 3$$

Proof. Let's prove this theorem by mathematical induction.

1. For $n = 3k$, $n\ mod\ 3 \equiv 0$, result is true for $n = 3$.
   $\gamma_{dev}(C_3 \otimes P_2) = 2.\lfloor\frac{3}{3}\rfloor + 3\ mod\ 3 = 2.\lfloor 1\rfloor + 0 = 2.1 + 0 = 2.$
   Our assumptation asserts that,
   $\gamma_{dev}(C_{3k} \otimes P_2) = 2.\lfloor\frac{3k}{3}\rfloor + 3k\ mod\ 3$
   $= 2.\lfloor\frac{3k}{3}\rfloor + 0 =$
   $= 2.\lfloor\frac{3k}{3}\rfloor.$
   We want to prove that for $n = 3(k+1) = 3k+3$, double edge-vertex domination number of cartesian product is
   $\gamma_{dev}(C_{3k+3} \otimes P_2) = 2.\lfloor\frac{3k+3}{3}\rfloor + (3k+3)\ mod\ 3$
   $= 2.\lfloor\frac{3k+3}{3}\rfloor + 0 = 2.\lfloor\frac{3k+3}{3}\rfloor.$

   The number of vertices of C_{3k+3} is 6 more than the number of vertices of C_{3k}. An edge dominates 6 vertices at most. Thus, at least 2 edges are required to dominate these 6 vertices twice. Therefore,
   $\gamma_{dev}(C_{3k+3} \otimes P_2) \geq \gamma_{dev}(C_{3k} \otimes P_2) + 2$
   $= 2.\ \lfloor\frac{3k}{3}\rfloor + 2$
   $= 2.\ \lfloor\frac{3k}{3}\rfloor + 2.1$
   $= 2.(\lfloor\frac{3k}{3}\rfloor + 1)$
   $\geq 2.\ \lfloor\frac{3k}{3} + 1\rfloor$
   $= 2.\ \lfloor\frac{3k}{3} + \frac{3}{3}\rfloor$
   $= 2.\ \lfloor\frac{3k+3}{3}\rfloor.$

2. For $n = 3k+1$, $n \ mod \ 3 \equiv 1$, result is true for $n = 4$.
   $\gamma_{dev}(C_4 \otimes P_2) = 2.\lfloor \frac{4}{3} \rfloor + 4 \ mod \ 3 = 2.1 + 1 = 3.$
   Our assumption asserts that,
   $\gamma_{dev}(C_{3k+1} \otimes P_2) = 2.\lfloor \frac{3k+1}{3} \rfloor + (3k+1) \ mod \ 3$
   $= 2.\lfloor \frac{3k+1}{3} \rfloor + 1.$
   We want to prove that for $n = 3(k+1)+1 = 3k+4$, double edge-vertex domination number of cartesian product is
   $\gamma_{dev}(C_{3k+4} \otimes P_2) = 2.\lfloor \frac{3k+4}{3} \rfloor + (3k+4) \ mod \ 3$
   $= 2.\lfloor \frac{3k+4}{3} \rfloor + 1.$
   The number of vertices of C_{3k+4} is 6 more than the number of vertices of C_{3k+1}. An edge dominates at most 6 vertices. Thus, at least 2 edges are required to dominate these 6 vertices twice. Therefore,
   $\gamma_{dev}(C_{3k+4} \otimes P_2) \geq \gamma_{dev}(C_{3k+1} \otimes P_2) + 2$
   $= 2. \left\lfloor \frac{3k+1}{3} \right\rfloor + (3k+1) \ mod \ 3 + 2$
   $= 2. \left\lfloor \frac{3k+1}{3} \right\rfloor + 1 + 2$
   $= 2. \left\lfloor \frac{3k+1}{3} \right\rfloor + 2 + 1$
   $= 2. \left\lfloor \frac{3k+1}{3} \right\rfloor + 2.1 + 1$
   $2.(\left\lfloor \frac{3k+1}{3} \right\rfloor + 1) + 1$
   $\geq 2. \left\lfloor \frac{3k+1}{3} + 1 \right\rfloor + 1$
   $= 2. \left\lfloor \frac{3k+1}{3} + \frac{3}{3} \right\rfloor + 1$
   $= 2. \left\lfloor \frac{3k+1+3}{3} \right\rfloor + 1$
   $= 2. \left\lfloor \frac{3k+4}{3} \right\rfloor + 2.$

3. For $n = 3k+2$, $n \ mod \ 3 \equiv 2$, result is true for $n = 5$.
   $\gamma_{dev}(C_5 \otimes P_2) = 2.\lfloor \frac{5}{3} \rfloor + 5 \ mod \ 3 = 2.1 + 2 = 4.$
   Our assumption asserts that,
   $\gamma_{dev}(C_{3k+2} \otimes P_2) = 2.\lfloor \frac{3k+2}{3} \rfloor + (3k+2) \ mod \ 3$
   $= 2.\lfloor \frac{3k+2}{3} \rfloor + 2.$
   We want to prove that for $n = 3(k+1)+2 = 3k+5$, double edge-vertex domination number of cartesian product is
   $\gamma_{dev}(C_{3k+5} \otimes P_2) = 2.\lfloor \frac{3k+5}{3} \rfloor + (3k+5) \ mod \ 3$
   $= 2.\lfloor \frac{3k+5}{3} \rfloor + 2.$
   The number of vertices of C_{3k+5} is 6 more than the number of vertices of C_{3k+2}. An edge dominates at most 6 vertices. Thus, at least 2 edges are required to dominate these 6 vertices twice. Therefore,
   $\gamma_{dev}(C_{3k+5} \otimes P_2) \geq \gamma_{dev}(C_{3k+2} \otimes P_2) + 2$
   $= 2. \left\lfloor \frac{3k+2}{3} \right\rfloor + (3k+2) \ mod \ 3 + 2$
   $= 2. \left\lfloor \frac{3k+2}{3} \right\rfloor + 2 + 2$
   $= 2. \left\lfloor \frac{3k+2}{3} \right\rfloor + 2.1 + 2$
   $= 2.(\left\lfloor \frac{3k+2}{3} \right\rfloor + 1) + 2$
   $\geq 2. \left\lfloor \frac{3k+2}{3} + 1 \right\rfloor + 2$
   $= 2. \left\lfloor \frac{3k+2}{3} + \frac{3}{3} \right\rfloor + 2$
   $= 2. \left\lfloor \frac{3k+2+3}{3} \right\rfloor + 2$
   $= 2. \left\lfloor \frac{3k+5}{3} \right\rfloor + 2.$

□

Theorem 4. *For a cycle with order $n \geq 3$ and path with order 3, double edge-vertex domination number of corona products of these are as follows,*

$$\gamma_{dev}\left(C_n \bigotimes \mathrm{P}_3\right) = n$$

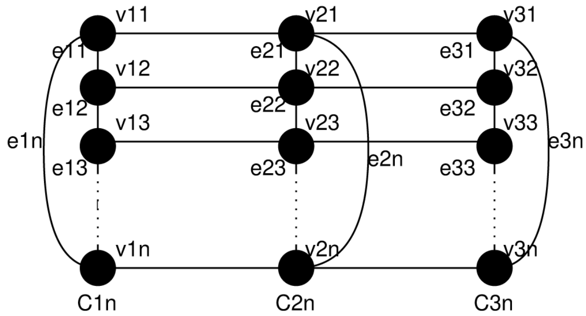

Figure 1: Cartesian product of Cn with P3

Proof. Let C_n be a cycle, C_{1n} be first copy of C_n, C_{2n} be second copy of C_n, C_{3n} be third copy of C_n in resulting graph after cartesian product. And let vertex sets of these copies to be, $V_1 = \{v_{11}, v_{12}, ..., v_{1n}\}$ be set of vertices of C_{1n}, $V_2 = \{v_{21}, v_{22}, ..., v_{2n}\}$ be set of vertices of C_{2n}, $V_3 = \{v_{31}, v_{32}, ..., v_{3n}\}$ be set of vertices of C_{3n} respectively. And let $E = \{e_{21}, e_{22}, ..., e_{2n}\}$ to be set of edges of C_{2n}. These edges and C_{2n} are joined to the second vertices set of P_n. The vertices with the largest neighborhood are belong to C_{2n}. Therefore, we start selection with the edges of C_{2n}. If we start with e_{21},then e_{21} dominates $v_{11}, v_{12}, v_{31}, v_{32}, v_{21}, v_{22}, v_{23}, v_{2n}$ vertices. If we continue with e_{22}, then e_{22} dominates $v_{12}, v_{13}, v_{32}, v_{33}, v_{21}, v_{22}, v_{23}, v_{24}$ vertices. So, $v_{12}, v_{32}, v_{21}, v_{22}, v_{23}$ vertices are dominated twice. Then, we continue with e_{23}, e_{23} dominates $v_{13}, v_{14}, v_{33}, v_{34}, v_{22}, v_{23}, v_{24}, v_{25}$. Hence, $v_{13}, v_{33}, v_{22}, v_{23}, v_{24}$ are dominated twice. If we continue in this way until e_{2n} edges, then all vetices are dominated twice. In this case we select n edges. Hence,

$$\gamma_{dev}\left(C_n \bigotimes \mathrm{P}_3\right) = n.$$

□

4 Conclusion

In graph theory, domination has an important role in the vulnerability analysis of communication networks modelled by graphs. There are many types of domination depending on structures of dominating sets. Domination has a very wide range of application side also. Both edge-vertex and vertex edge domination are concepts on which many researchers are interested in recent works. Depending on the structure of the network, sometimes any break down on links have more importance. In this sense, parameters on edge dominating sets are widely studied. In this paper we investigated double edge-vertex domination(Kılıç & Aylı, 2019; Kılıç & Aylı, 2020) number of graphs under corona operations. For future work we are planning to work on other structural operations on graphs. As two open problems on the subject we can say that are, first: upper and lower bounds of double edge vertex domination number of trees can be generalized and second: an algorithm can be determined to find this number on some specific classes of graphs.

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